HW 9 Sum and Difference Identities (14.4) **Name** Key

I will be able to use sum and difference identities.

Sum and Difference Identities

$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\cos(A+B)=\cos A\cos B-\sin A\sin B$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$

Examples:

A.) Find **sin (15°)**

*check w/ calculator

$\sin(45°-30°) = \sin(45)\cos(30) - \cos(45)\sin(30)$

$\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$ unit circle

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

Find the exact value for each expression.

1.) sin (30° + 45°)

$\sin(30)\cos(45)+\cos(30)\sin(45)$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

2.) sin (30° + 135°)

$\sin(30)\cos(135)+\cos(30)\sin(135)$

$\frac{1}{2}\cdot\frac{-\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{-\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

3.) cos (30° + 45°)

$\cos(30)\cos(45) - \sin(30)\sin(45)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

4.) sin (210° - 315 °)

$\sin(210)\cos(315)-\cos(210)\sin(315)$

$\frac{-1}{2}\cdot\frac{\sqrt{2}}{2} - \left(\frac{-\sqrt{3}}{2}\cdot\frac{-\sqrt{2}}{2}\right)$

$\frac{-\sqrt{2}}{4}-\frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$

5.) cos (150° - 45°)

$\cos(150)\cos(45)+\sin(150)\sin(45)$

$\frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{-\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

6.) sin (240° - 315°)

$\sin(240)\cos(315) - \cos(240)\sin(315)$

$\frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \left(\frac{-1}{2}\cdot\frac{-\sqrt{2}}{2}\right)$

$\frac{-\sqrt{6}}{4}-\frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

7.) sin (105°) $=\sin(60+45)$

$\sin(60)\cos(45)+\cos(60)\sin(45)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

8.) sin (165°) $=\sin(30+135)$

$\sin(30)\cos(135)+\cos(30)\sin(135)$

$\left(\frac{1}{2}\cdot\frac{-\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$\frac{-\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \frac{-\sqrt{2}+\sqrt{6}}{4}$

9.) sin (75°) $=\sin(30+45)$

$\sin(30)\cos(45)+\cos(30)\sin(45)$

$\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

10.) cos (225°) unit circle

$\frac{-\sqrt{2}}{2}$

11.) cos (-225°) $= \frac{-\sqrt{2}}{2}$

360

135°

unit circle

12.) sin (285°) $=\sin(240+45)$

$\sin(240)\cos(45)+\cos(240)\sin(45)$

$\frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \left(\frac{-1}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$\frac{-\sqrt{6}}{4}+\frac{-\sqrt{2}}{4}$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

Double and Half-Angle Identities (14.5)

Name Key

I will be able to use sum and difference identities.

Double Angles

$\sin(2A) = 2\sin(A)\cos(A)$

$\cos(2A) = \cos^2(A) - \sin^2(A)$

Half Angles

$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$

$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$

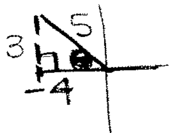
± depends on value for sine or cosine

① Quadrant
② Draw Triangle
③ Use Formulas

Examples:

Use the information to find the exact value of sin 2θ and cos2θ.

A.) $90 \le \theta \le 180;\ \sin\theta = \frac{3}{5}$ II



$5^2 - 3^2 = 16$

$\sqrt{16} = 4$

a) $\sin(2\theta) = 2\sin\theta\cos\theta$

$\frac{2}{1}\cdot\frac{3}{5}\cdot\frac{-4}{5} = \frac{-24}{25}$

$\cos\theta = \frac{-4}{5}$

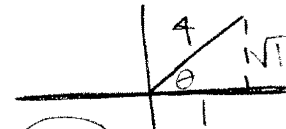
b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

partner

B.) $0 \le \theta \le 90;\ \cos\theta = \frac{1}{4}$ I

$\sin = \sqrt{15}/4$



$4^2 - 1^2 = b^2$

$16 - 1 = b^2$

$15 = b^2$

a) $\sin(2\theta) = 2\sin\theta\cos\theta$

$\frac{2}{1}\cdot\frac{\sqrt{15}}{4}\cdot\frac{1}{4} = \frac{2\sqrt{15}}{16}$

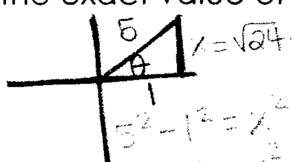
b) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{1}{4}\right)^2 - \left(\frac{\sqrt{15}}{4}\right)^2$

$\frac{1}{16} - \frac{15}{16} = \frac{-14}{16} = \frac{-7}{8}$

Use the information to find the exact value of sin ½θ and cos ½θ.

C.) $0 \le \theta \le 90;\ \cos\theta = \frac{1}{5}$ I



$x = \sqrt{24}$ partner

$5^2 - 1^2 = x^2$

$25 - 1 = x^2$

$24 = x^2$

$\sin\frac{\theta}{2} = +\sqrt{\frac{1-1/5}{2}}$ $\sin\theta = \frac{\sqrt{24}}{5}$

$= +\sqrt{\frac{4/5}{2}} = \sqrt{4/10}$

$\cos\frac{\theta}{2} = +\sqrt{\frac{1+1/5}{2}} = \sqrt{\frac{6/5}{2}} = \sqrt{\frac{6}{10}}$

D.) $90 \le \theta \le 180;\ \sin\theta = \frac{5}{6}$ II

$6^2 - 5^2 = x^2$

$36 - 25 = x^2$

$11 = x^2$

$\cos = \frac{-\sqrt{11}}{6}$

$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\sqrt{11}/6}{2}}$

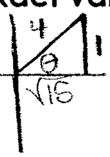
$= \sqrt{\frac{6/6 - \sqrt{11}/6}{2}} = \sqrt{\frac{6-\sqrt{11}}{12}}$

$\cos\frac{\theta}{2} = -\sqrt{\frac{6/6 + \sqrt{11}/6}{2}} = \sqrt{\frac{6+\sqrt{11}}{12}}$

Use the information to find the exact value of sin 2θ and cos2θ.

1.) $0 \le \theta \le 90;\ \sin\theta = \frac{1}{4}$ I

$\cos\theta = \frac{\sqrt{15}}{4}$



$4^2 - 1^2 = x^2$

$16 - 1 = 15$

$\sqrt{15} = x$

a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\cdot\frac{1}{4}\cdot\frac{\sqrt{15}}{4} = \frac{2\sqrt{15}}{16} = \boxed{\frac{\sqrt{15}}{8}}$

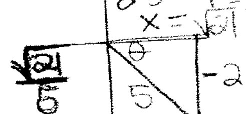
b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2$

$\frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \boxed{\frac{7}{8}}$

2.) $270 \le \theta \le 360;\ \sin\theta = -\frac{2}{5}$ IV

$\cos\theta = \frac{\sqrt{21}}{5}$



$5^2 - (-2^2) =$

$25 - 4 =$

$x = \sqrt{21}$

a) $\sin 2\theta = 2\sin\theta\cos\theta$

$2\cdot\left(\frac{-2}{5}\right)\cdot\frac{\sqrt{21}}{5} = \frac{-4\sqrt{21}}{25}$

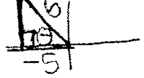
b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{-2}{5}\right)^2$

$\frac{21}{25} - \frac{4}{25} = \frac{17}{25}$

Use the information to find the exact value of sin ½θ and cos ½θ.

3.) $90 \le \theta \le 180;\ \cos\theta = \frac{-5}{6}$ II



a) $\sin\frac{\theta}{2} = +\sqrt{\frac{1-(-5/6)}{2}}$

$= \sqrt{\frac{11/6}{2}} = \sqrt{\frac{11}{12}}$

b) $\cos\frac{\theta}{2} = +\sqrt{\frac{1+(-5/6)}{2}}$

$= \sqrt{\frac{6/6 - 5/6}{2}} = \sqrt{\frac{1/6}{2}} = \sqrt{\frac{1}{12}}$

4.) $180 \le \theta \le 270;\ \cos\theta = \frac{-\sqrt{5}}{3}$ III

90 135

$\sin\frac{\theta}{2} = -\sqrt{\frac{1-(-\sqrt{5}/3)}{2}} = -\sqrt{\frac{3/3 + \sqrt{5}/3}{2}}$

$= -\sqrt{\frac{3+\sqrt{5}}{6}}$

$\cos\frac{\theta}{2} = +\sqrt{\frac{1+(-\sqrt{5}/3)}{2}} = \sqrt{\frac{3/3 - \sqrt{5}/3}{2}} = \sqrt{\frac{3-\sqrt{5}}{6}}$